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Hopf Galois extensions, triangular structures, and Frobenius Lie algebras in prime characteristic

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Introduction

The final goal of this paper is to introduce certain finite dimensional Hopf algebras associated with restricted Frobenius Lie algebras over a field of characteristic $p > 0$. The antipodes of these Hopf algebras have order either $2p$ or 2, and in the minimal dimension p^2 there exists just one Hopf algebra in this class which coincides with an example due to Radford [35] of a Hopf algebra with a nonsemisimple antipode (the first example of this kind in [44] does not quite fit into our scheme). Another feature of the Hopf algebras under consideration is that they admit triangular structures of maximal rank, so that they are isomorphic to their dual Hopf algebras taken with the opposite multiplications. Regarding the algebra structure alone, these Hopf algebras are isomorphic to the restricted enveloping algebras of the corresponding Lie algebras (such isomorphisms are not canonical). In particular, they are never semisimple. It should be mentioned that all semisimple cosemisimple triangular Hopf algebras over any algebraically closed field were classified by Etingof and Gelaki [12].

The Hopf algebras we deal with are instances of transformations in Hopf Galois extensions discovered not so long ago. The Hopf Galois theory originated in the work of Chase and Sweedler [5] and received a full treatment by Kreimer and Takeuchi [21]. We recall the notion of a Hopf Galois algebra in Section 1 of the paper. Suppose that H is a finite dimensional Hopf algebra over a field and A is a left H-module algebra which is H^*-Galois with respect to the corresponding right H^*-comodule structure. A construction of Schauenburg [38] generalizing a special case of commutative A and H^* considered

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